BC Calc - Partial Fractions Practice ws

① $\int \frac{1}{x^2+2x}\,dx$

$\int \frac{1}{x(x+2)}\,dx$ $\qquad \frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$= \int \frac{1}{2x} - \frac{1}{2(x+2)}\,dx$ $\qquad x=0 \quad A=\frac{1}{2}$ $\qquad x=-2 \quad B=-\frac{1}{2}$

$= \frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$

or $= \frac{1}{2}\ln\left|\frac{x}{x+2}\right| + C$ (log properties)

② $\int \frac{x}{x^2-2x-3}\,dx$

$= \int \frac{x}{(x-3)(x+1)}\,dx$ $\qquad \frac{x}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$

$= \int \left(\frac{3}{4(x-3)} + \frac{1}{4(x+1)}\right)dx$ $\qquad x=3 \quad A=\frac{3}{4}$ $\qquad x=-1 \quad B=\frac{1}{4}$

$= \frac{3}{4}\ln|x-3| + \frac{1}{4}\ln|x+1| + C$

or $\frac{1}{4}\ln\left|(x-3)^3\cdot(x+1)\right| + C$

(log properties)

⇒

③ $\int \frac{x^3}{x^2-2x-3}\,dx$

$$x^2-2x-3 \;\overline{)\; x^3+0x^2+0x+0}\;\; x+2$$

$$x^3-2x^2-3x$$

$$2x^2+3x+0$$

$$2x^2-4x-6$$

$$7x+6$$

$$= \int \left(x+2+\frac{7x+6}{x^2-2x-3}\right)dx$$

$$= \frac{1}{2}x^2+2x+\int \frac{7x+6}{(x-3)(x+1)}$$

$$\frac{7x+6}{(x-3)(x+1)} = \frac{A}{x-3}+\frac{B}{x+1}$$

$$= \frac{1}{2}x^2+2x+\frac{27}{4}\ln|x-3|+\frac{1}{4}\ln|x+1|+C$$

$x=3$ $\quad A=\frac{27}{4}$ $\qquad$ $x=-1$ $\quad B=\frac{1}{4}$

④ $\int \frac{2x^2}{x^2+1}\,dx$

$$x^2+1 \;\overline{)\; 2x^2+0x+0}\;\; 2$$

$$2x^2+2$$

$$-2$$

$$= \int \left(2-\frac{2}{x^2+1}\right)dx$$

$$= 2x-2\tan^{-1}(x)+C$$

(5) $\int \frac{\cos x \, dx}{\sin^2 x + \sin x - 6}$ $u = \sin x$, $du = \cos x \, dx$

$= \int \frac{1}{u^2 + u - 6} \, du$

$= \int \frac{1}{(u+3)(u-2)} \, du$ $\frac{1}{(u+3)(u-2)} = \frac{A}{u+3} + \frac{B}{u-2}$

$u = -3$ $u = 2$

$= -\frac{1}{5} \ln|u+3| + \frac{1}{5} \ln|u-2| + C$ $A = -\frac{1}{5}$ $B = \frac{1}{5}$

$= -\frac{1}{5} \ln|\sin x + 3| + \frac{1}{5} \ln|\sin x - 2| + C$

(6) $\int \frac{5x-13}{(x-3)(x-2)} \, dx$ $\frac{5x-13}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$

$x = 3$ $x = 2$

$= \int \left(\frac{2}{x-3} - \frac{3}{x-2} \right) dx$ $A = 2$ $B = 3$

$= 2 \cdot \ln|x-3| + 3 \ln|x-2| + C$

(7) $\int \frac{dx}{1+4x^2}$

$\int \frac{1}{1+(2x)^2} \, dx$ $u = 2x$, $du = 2 \, dx$

$= \frac{1}{2} \tan^{-1}(2x) + C$

(8) $\int \frac{x+3}{2x^3-8x}\,dx$

$= \frac{1}{2}\int \frac{x+3}{x(x+2)(x-2)}\,dx \qquad \frac{x+3}{2x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$x=0$	$x=-2$	$x=2$
$A=-\frac{3}{4}$	$B=\frac{1}{8}$	$C=\frac{5}{8}$

$= \frac{1}{2}\left(-\frac{3}{4}\ln|x| + \frac{1}{8}\ln|x+2| + \frac{5}{8}\ln|x-2|\right) + C$

$= -\frac{3}{8}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{5}{16}\ln|x-2| + C$

(9) $\frac{dy}{dx} = \frac{1}{(x-2)(x-1)}$

$\int dy = \int \frac{1}{(x-2)(x-1)}\,dx \qquad \frac{1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$

$x=2$	$x=1$
$A=1$	$B=-1$

$y = \ln|x-2| - \ln|x-1| + C$

$y = \ln\left|\frac{x-2}{x-1}\right| + C$

$0 = \ln\frac{1}{2} + C$

$C = -\ln\frac{1}{2} = \ln 2$

$y = \ln\left|\frac{x-2}{x-1}\right| + \ln 2 = \ln\left|\frac{2(x-2)}{x-1}\right|$